Shear wave propagation in magneto poroelastic medium sandwiched between self-reinforced poroelastic medium and poroelastic half space

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Abstract

Purpose – The purpose of this paper is to investigate the effect of reinforcement, inhomogeneity and initial stress on the propagation of shear waves. The problem consists of magneto poroelastic medium sandwiched between self-reinforced medium and poroelastic half space. Using Biot's theory of wave propagation, the frequency equation is obtained.

Design/methodology/approach – Shear wave propagation in magneto poroelastic medium embedded between a self-reinforced medium and poroelastic half space is investigated. This particular setup is quite possible in the Earth crust. All the three media are assumed to be inhomogeneous under initial stress. The significant effects of initial stress and inhomogeneity parameters of individual media have been studied.

Findings – Phase velocity is computed against wavenumber for various values of self-reinforcement, heterogeneity parameter and initial stress. Classical elasticity results are deduced as a particular case of the present study. Also in the absence of inhomogeneity and initial stress, frequency equation is discussed. Graphical representation is made to exhibit the results.

Originality/value – Shear wave propagation in magneto poroelastic medium embedded between a self-reinforced medium, and poroelastic half space are investigated in presence of initial stress, and inhomogeneity parameter. For heterogeneous poroelastic half space, the Whittaker's solution is obtained. From the numerical results, it is observed that heterogeneity parameter, inhomogeneity parameter and reinforcement parameter have significant influences on the wave characteristics. In addition, frequency equation is discussed in absence of inhomogeneity and initial stress. For the validation purpose, numerical results are also computed for a particular case.

Keywords Shear wave, Self-reinforcement, Magneto poroelastic, Half-space, Inhomogeneity, Initial stress

Paper type Research paper

1. Introduction
Study of mechanical behavior of self-reinforced material has a huge importance in the domains such as structural engineering and geomechanics. Self-reinforced materials attract many

investigators in the field of engineering applications due to their superiority over other structural materials in the applications where high strength and stiffness and light weight components are required. Most of the structures on or near the surface of the Earth are self-reinforced and are constructed in such a manner that the effect of earthquake or similar disturbances causes minimum destruction. Many elastic fiber-reinforced composite materials are strongly anisotropic in nature. There are many materials for example, the Earth's crust and mantle which are inhomogeneous. The Earth crust contains some hard and soft rocks that exhibit both self-reinforcement property and inhomogeneity. As a result of stress developed within the crust, deformation takes place along with fracture. These fractures release large amount of energy which give rise to elastic waves travel and finally reach to surface. Moreover, the Earth crust made of diversity of igneous, metamorphic and sedimentary rocks. These rocks are capable to generate magnetic field due to presence of some minerals like iron, nickel and cobalt in them. For the said reasons, these rocks can be treated as magneto poroelastic solids. An increasing attention is being devoted to the interactions between magnetic and strain fields. Belfield *et al.* (1983) introduced the continuous reinforcement at every point of an elastic solid. Vardoulakis (1984) studied the propagation of torsional surface waves in heterogeneous elastic media. Propagation of shear waves and transverse surface waves in self–reinforced magneto elastic bodies is investigated by Verma (1986) and Verma *et al.* (1988). Chattopadhyay and Chaudhary (1995) discussed the propagation of magneto elastic shear waves in an infinite self-reinforced plate. Chaudhary *et al.* (2006) reveal that reflection and transmission coefficients are strongly influenced by the reinforcement parameters, and angle of incidence of the incident wave. Chattopadhyay *et al.* (2010) investigated the dispersion of shear wave in multilayered magneto elastic self-reinforced media. Effects of point source, self-reinforcement and heterogeneity on the propagation of magneto elastic shear waves are presented in Chattopadhyay *et al.* (2011). Samal and Chattaraj (2011) discussed the surface wave propagation in fiber-reinforced anisotropic elastic layer between liquid saturated porous half space and uniform liquid layer. Propagation of shear waves in an irregular self-reinforcement layer, the combined effects of reinforcement, magnetic field and irregularity are studied by Chattopadhyay and Kumar (Chattopadhyay and Singh, 2012). Chattopadhyay *et al.* (2012) discussed the torsional surface waves in a self-reinforced medium over a heterogeneous half space. Using the dual reciprocity boundary element method (DRBEM), Fahmy (2012a, 2012b, 2012c, 2012d) studied the transient magneto-thermo-viscoelastic stresses in a non-homogeneous anisotropic solid subjected to a heat source. The effect of a moving heat source on the transient magneto-thermo-visco-elastic stresses in a non-homogeneous anisotropic solid is investigated (Fahmy, 2012a, 2012b, 2012c, 2012d). Fahmy (2012a, 2012b, 2012c, 2012d) discussed the generalized magneto thermoelasticity problem of rotating anisotropic viscoelastic functionally graded solids. Generalized magneto-thermo viscoelastic problem of rotating functionally graded materials are investigated by Fahmy (2012a, 2012b, 2012c, 2012d) and Fahmy (2013). Abd-Alla *et al.* studied the surface waves propagation in fiber-reinforced anisotropic elastic media subjected to gravitational field (Abd-Alla *et al.*, 2013). Chattaraj and Samal studied the propagation of Love waves in fiber-reinforced layer lying over a gravitating anisotropic porous half space (Chattaraj and Samal, 2013). From Chattaraj and Samal (2013), authors concluded that the transverse and longitudinal rigidity of reinforced material, and porosity of half space have a significant effect on wave propagation. Chatterjee *et al.* (2015) studied the propagation of shear waves in visco-elastic heterogeneous layer overlying on initially stressed half space. Effect of reinforcement on half-space is discussed (Singh *et al.*, 2016). Tong *et al.* (2016) investigated wave propagation characteristics in fluid saturated porous materials in the framework of nonlocal Biot's theory. By considering the interface between self-reinforced medium and half space, effect of reinforcement and heterogeneity on the propagation

of torsional surface waves is discussed by Paswan et al. (2017). Effect of gravity and initial stress on propagation of torsional surface waves in heterogeneous medium is studied (Mukhopadhyay et al., 2017). The reflection of plane waves in a rotating magneto elastic fibre–reinforced semi space is investigated by Roy et al. (2017). From Roy et al. (2017), it is clear that magnetic field, rotation, fibre-reinforcement and surface stress play vital role in wave propagation, particularly, in reflection of plane waves. Using Biot's theory (Biot, 1956), shear wave propagation in magneto poroelastic dissipative isotropic medium sandwiched between two poroelastic half spaces is studied (Balu et al., 2017). In Balu et al. (2017), wave characteristics are computed at the fixed heterogeneous and magnetic poroelastic coupling factor. Tong et al. (2018) studied the Rayleigh wave propagation in porous fluid saturated materials and concluded that the nonlocal parameter does not have significant influence on the characteristics of Rayleigh waves within a low frequency range when compared to that of classical Biot theory. Effect of several parameters of reinforced sandwich porous plates such as aspect ratios, volume fraction, types of reinforcement and thickness of plate on bending is investigated by Medani et al. (2019). The nonlinear static, buckling and vibration analysis of viscoelastic micro-composite beam reinforced by various distributions of boron nitride nanotube (BNNT) is studied by using finite element method (FEM) (Alimirzaei et al., 2019). Vibration analysis of functionally graded micro beams is studied by Tlidji et al. (2019). Boukhlif et al. (2019) made dynamic investigation of functionally graded (FG) plates resting on an elastic foundation using a simple quasi-3D HSPT in which the stretching effect is considered. Some recent papers (Karami et al., 2019a, 2019b; Karami et al., 2019a, 2019b) are reported in the field of the functionally graded nanoplates reinforced with graphene nanoplates and nanoshell in magnetic field. In the above-mentioned papers, the authors have not studied the wave propagation in magneto poroelastic medium sandwiched between self-reinforced medium and poroelastic half space. Hence, in the present work, the same is investigated. This particular setup is quite possible in the Earth's crust. All the three media are assumed to be inhomogeneous under initial stress. The significant effects of initial stress, and inhomogeneity parameters of the individual media have been studied.

This paper is organized as follows. In Section 2, formulation of the problem is given. The solution of the problem is discussed in Section 3. Boundary conditions and frequency equation are presented in Section 4. In Section 5, particular cases are discussed. Numerical results are described in Section 6. Finally, conclusion is given in Section 7.

2. Formulation of the problem

Consider the magneto poroelastic medium sandwiched between inhomogeneous self-reinforced medium and poroelastic half space, all assumed to be under initial stress. The origin O is considered in a manner such that it separates the sandwiched layer from upper and lower half-spaces. The horizontal axis is assumed to be along r-axis, and z-axis is in downward direction as shown in Figure 1. From the figure, it is clear that upper self-reinforcement layer M_1 (say), magneto poroelastic layer M_2 (say), and poroelastic half space M_3 (say) occupy the spaces $(-h_2 < z < -h_1).(-h_1 < z < 0)$ and $(z > 0)$, respectively. Equations of motion for shear deformation in the isotropic poroelastic medium in absence of dissipation are (Biot, 1956)

$$N\left(\nabla^2 - \frac{1}{r^2} + \frac{2}{r^2}\frac{\partial}{\partial\theta}\right)v + (A+N)\frac{\partial e}{\partial r} + Q\frac{\partial\varepsilon}{\partial r} = \frac{\partial^2}{\partial t^2}(\rho_{11}v + \rho_{12}V),$$
$$\frac{Q}{r}\frac{\partial e}{\partial\theta} + \frac{R}{r}\frac{\partial\varepsilon}{\partial\theta} = \frac{\partial^2}{\partial t^2}(\rho_{12}v + \rho_{22}V). \tag{1}$$

In equation (1), v and V are the displacement components of solid and fluid, respectively, $P = A + 2N,Q$, and R are all poroelastic constants, e and ε are the dilatations of solid and

$(\sigma_{\theta z})_{M_1} = 0$

Self-reinforced medium (M_1)

h_2

$(\sigma_{\theta z})_{M_1} = (\sigma_{\theta z})_{M_2}, v_1 = v_2,$

Magneto poroelastic medium (M_2)

h_1

P_{10} P_{1o}

P_{20} P_{20}

$(\sigma_{\theta z})_{M_2} = (\sigma_{\theta z})_{M_3}, v_2 = v_3.$

Z = 0

Poroelastic medium (M_3)
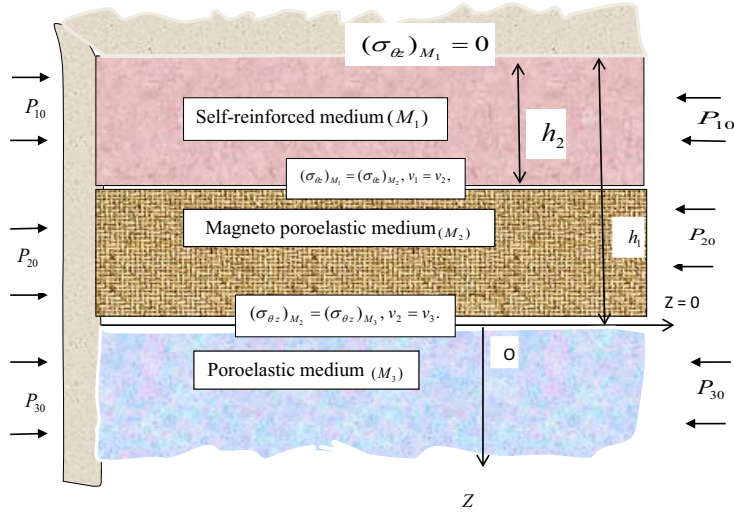
O

P_{30} P_{30}

Z

Figure 1.
Geometry of the
problem

fluid. The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} satisfy the relations $\rho_1 = \rho_{11} + \rho_{12} = (1-\beta)\rho_s$, $\rho_2 = \rho_{12} + \rho_{22} = \beta \rho_f, \rho = \rho_1 + \rho_2 = \rho_s + \beta(\rho_f - \rho_s)$. Here, ρ_1 and ρ_2 are masses of solid and fluid per unit volume of aggregate, ρ is total mass of solid-fluid aggregate per unit volume. ρ_s and ρ_f are mass densities of the solid and fluid, respectively. β is the porosity of the aggregate. The solid stresses σ_{ij}, and fluid pressure s are given by (Biot, 1956):

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij}, \quad (i,j=1,2,3), \quad s = Qe + R\varepsilon. \tag{2}$$

In equation (2), e'_{ij}s are strain components, and δ_{ij} is the well-known Kronecker delta function.

3. Solution of the problem
The solutions for three medium are presented in the following sub sections.

3.1 Upper self-reinforced medium$(-h_2 < z < -h_1)$
Let (u_1, v_1, w_1) and (U_1, V_1, W_1) be the displacement components of solid and fluid, respectively, in the upper self-reinforced medium. For the shear vibrations, displacement components are $u_1 = w_1 = 0, v_1 = v_1(r,z,t)$, and $U_1 = W_1 = 0, V_1 = V_1(r,z,t)$. The stress tensor for a self-reinforced medium along the preferred direction $\overrightarrow{a}$ is given below (Chattopadhyay et al., 2011).

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_l e_{kl}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$
$$+ \beta a_k a_m e_{km} a_i a_j,$$

Here, $i,j,k,l,m = 1,2,3$, and the coefficients $\lambda, \alpha, \beta, \mu_T, \mu_L$ are the elastic constants with the dimension of stress, μ_T and μ_L are the transverse and longitudinal shear modulus, respectively, the vector $\overrightarrow{a} = (a_1, a_2, a_3)^T$ is the preferred unit direction of reinforcement so

that $a_1^2 + a_2^2 + a_3^2 = 1$. In the plane under consideration, $\overrightarrow{a}_i = (\sin\phi, 0, \cos\phi)^T$, ϕ is the angle of inclination to the radial axis. For torsional vibrations, the strain components are as follows:

$$e_{rr} = e_{\theta\theta} = e_{zz} = e_{zr} = 0, e_{\theta z} = \frac{1}{2}\frac{\partial v}{\partial z}, \text{ and } e_{r\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right).$$

Using the above in equation (2), the non-zero stress components are given by:

$$\sigma_{r\theta} = \mu_T\left[S_1\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) + S_2\frac{\partial v}{\partial z}\right], \sigma_{\theta z} = \mu_T\left[S_2\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) + S_3\frac{\partial v}{\partial z}\right]. \tag{3}$$

In equation (3), $S_1 = \left(1 + (\mu' - 1)a_1^2\right)$, $S_2 = (\mu'-1)\ a_1a_3$, $S_3 = 1 + (\mu' - 1)a_3^2$, $\mu' = \frac{\mu_L}{\mu_T}, \mu_L = \mu_{L_0}e^{az}, \mu_T = \mu_{T_0}e^{az}$.

In this case, the equations of motion are:

$$\frac{\partial}{\partial r}(\sigma_{r\theta}) + \frac{\partial}{\partial z}(\sigma_{\theta z}) + \frac{2}{r}(\sigma_{r\theta}) - \frac{\partial}{\partial z}(P_1e_{\theta z}) = \frac{\partial^2}{\partial t^2}(\rho_{11}v_1 + \rho_{12}V_1),$$

$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12}v_1 + \rho_{22}V_1). \tag{4}$$

Because of the inhomogeneity with respect to the z coordinate the following are assumed in equation (4), P_1 denotes the initial stress, and $P_1 = P_{10}e^{az}, \rho_{11} = \rho_{110}e^{az}, \rho_{12} = \rho_{120}e^{az}, \rho_{22} = \rho_{220}e^{az}$, here a is a constant, $P_{10}, \rho_{110}, \rho_{120}, \rho_{220}$, are the values of their respective parameters at $z = 0$ For harmonic waves, the displacement component $v_1(z)$ is assumed as:

$$v_1(z) = f_1(z)J_1(kr)e^{i\omega t}, \tag{5}$$

Here, $k, c, \omega(=kc)$ and $J_1(.)$ are wavenumber, wave velocity, circular frequency and Bessel's function of first order and first kind, respectively. From equations (3), (4) and (5), the following equation is obtained:

$$\frac{d^2f_1}{dz^2} + E\frac{df_1}{dz} + Ff_1 = 0. \tag{6}$$

In equation (6),

$$E = \frac{M_{20}kJ_1'(kr)}{M_{40}J_1(kr)} + \frac{M_{60}}{M_{40}},$$

$$F = \frac{M_{10}J_1''(kr)k^2}{M_{40}J_1(kr)} + \frac{M_{10}J_1(kr)k}{rM_{40}J_1(kr)} + \frac{aM_{20}J_1'(kr)k}{M_{40}J_1(kr)} - \frac{M_{10}}{r^2M_{40}} - \frac{aM_{20}}{rM_{40}}$$

$$+ \omega^2\left(\frac{\rho_{110}\rho_{220} - \rho_{120}^2}{\rho_{220}\mu_{T_0}M_{40}J_1(kr)}\right),$$

$M_{10} = 1 + \left(\mu_0' - 1\right)a_1^2$, $M_{20} = \left(\mu_0' - 1\right)a_1a_3$, $M_{60} = (M_{20}/r) + aM_{40}$, $M_{40} = (M_{30} - \xi_{10})$, $M_{30} = 1 + \left(\mu_0' - 1\right)a_3^2$ $\xi_{10} = P_{10}/2\mu_{T_0}$.

Substitution of the solution of equation (6) in equation (5) gives:

$$v_1(z) = \exp(-Ez/2)\big(C_1\cos(\chi_1 z) + C_2\sin(\chi_1 z)\big)J_1(kr)e^{i\omega t}. \tag{7}$$

In equation (7), $\chi_1^2 = F - \frac{E^2}{4}$, C_1, C_2 are arbitrary constants. The pertinent stress component $\sigma_{\theta z}$ is given by:

$$
\begin{aligned}
(\sigma_{\theta z})_{M_1} = C_1 \bigg(& \mu_T S_2 k\sin(\chi_1 z)J_1'(kr) - S_2\mu_T\sin(\chi_1 z)\frac{J_1(kr)}{r} + S_3\cos(\chi_1 z)\chi_1 J_1(kr) \\
& -S_3\frac{E}{2}\sin(\chi_1 z)J_1(kr) \bigg) + C_2\bigg(\mu_T S_2 k\cos(\chi_1 z)J_1'(kr) - \mu_T S_2\cos(\chi_1 z)\frac{J_1(kr)}{r} \\
& +S_3\sin(\chi_1 z)\chi_1 J_1(kr) - S_3\frac{E}{2}\cos(\chi_1 z)J_1(kr) \bigg).
\end{aligned}
$$

3.2 The sandwiched magneto poroelastic medium($-h_1 < z < 0$)

Let (u_2, v_2, w_2) and (U_2, V_2, W_2) be the solid and fluid displacement components, respectively, in the sandwiched magneto poroelastic medium. For shear wave, equation (1) takes the following form:

$$
\begin{aligned}
&\frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial\sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + P_2\frac{\partial w_r}{\partial z} + \big(\overrightarrow{J}\times\overrightarrow{B}\big)_\theta = \frac{\partial^2}{\partial t^2}(\rho_{1100}v_2 + \rho_{1200}V_2),\\
&0 = \frac{\partial^2}{\partial t^2}(\rho_{1200}v_2 + \rho_{2200}V_2).
\end{aligned}
\tag{8}
$$

In equation (8), $\big(\overrightarrow{J}\times\overrightarrow{B}\big)_\theta$ is the θ component of electromagnetic force, $\overrightarrow{J}$ is the electric current density and $\overrightarrow{B}$ is the magnetic induction vector. The Maxwell's equations of the electromagnetic field are (Balu *et al.*, 2017):

$$\overrightarrow{\nabla}\cdot\overrightarrow{B} = 0, \overrightarrow{\nabla}\times\overrightarrow{E} = -\frac{\partial\overrightarrow{B}}{\partial t}, \overrightarrow{\nabla}\times\overrightarrow{H} = \overrightarrow{J}, \overrightarrow{B} = \mu_e\overrightarrow{H}, \overrightarrow{J} = \sigma\left(\overrightarrow{E} + \frac{\partial v}{\partial t}\times\overrightarrow{B}\right), \tag{9}$$

here, $\overrightarrow{E}$ is the induced electric field, $\overrightarrow{H}$ is the magnetic field consisting both primary and induced magnetic field, μ_e and σ are the induced permeability and conduction coefficient, respectively. The Maxwell stress tensor $\big(\tau_{ij}^0\big)^{M_x} = \mu_e(H_i p_j - H_j p_i - H_k p_k\delta_{ij})$. Let $\overrightarrow{H} = (H_r, H_\theta, H_z)$ and change in magnetic field be (P_1, P_2, P_3) If the displacement current is absent, then equation (9) reduces to:

$$\nabla^2 H = \mu_e\sigma\left(\frac{\partial\overrightarrow{H}}{\partial t} + \overrightarrow{\nabla}\times\left(\frac{\partial v_2}{\partial t}\times\overrightarrow{H}\right)\right). \tag{10}$$

Axial component of equation (10)

$$\frac{\partial H_\theta}{\partial t} = \frac{\partial}{\partial r}\left(H_r\frac{\partial v_2}{\partial t}\right) + \frac{\partial}{\partial z}\left(H_z\frac{\partial v_2}{\partial z}\right). \tag{11}$$

It is assumed that the primary magnetic field is uniform throughout the space. It is clear that there is no perturbation in H_r and H_z, but there is perturbation p_2 in H_θ Then $H_r = H_{01}, H_\theta =$

$H_{02} + p_2$, and $H_z = H_{03}$ where H_{01}, H_{02}, H_{03} are components of the initial magnetic field $\overrightarrow{H}_0$. As $\overrightarrow{H} = (H_0\cos\phi, p_2, H_0\sin\phi)$, where $H_0 = |\overrightarrow{H}_0|$ and ϕ is the angle at which wave crosses the magnetic field. Thus, $\overrightarrow{H}$ can be expressed as:

$$\overrightarrow{H} = (H_0\cos\phi, p_2, H_0\sin\phi). \tag{12}$$

Here, initially it is assumed that p_2 is zero. Then equations (11) and (12) give:

$$\frac{\partial p_2}{\partial t} = \frac{\partial}{\partial r}\left(H_0\cos\phi \frac{\partial v_2}{\partial t}\right) + \frac{\partial}{\partial z}\left(H_0\sin\phi \frac{\partial v_2}{\partial t}\right)$$

Integration of above equation with respect to t gives:

$$p_2 = H_0\cos\phi \frac{\partial v_2}{\partial r} + H_0\sin\phi \frac{\partial v_2}{\partial z}. \tag{13}$$

Equation (9) and the relation $\nabla(H^2/2) = -(\overrightarrow{\nabla} \times \overrightarrow{H}) \times \overrightarrow{H} + (\overrightarrow{H}.\overrightarrow{\nabla}).\overrightarrow{H}$ give the electromagnetic force as given below:

$$\left(\overrightarrow{J} \times \overrightarrow{B}\right) = \mu_e\left(-\overrightarrow{\nabla}(H^2/2) + (\overrightarrow{H}.\overrightarrow{\nabla})\overrightarrow{H}\right).$$

In this case, the components of $\left(\overrightarrow{J} \times \overrightarrow{B}\right)$ are $\left(\overrightarrow{J} \times \overrightarrow{B}\right)_r = 0$, $\left(\overrightarrow{J} \times \overrightarrow{B}\right)_z = 0$, and

$$\left(\overrightarrow{J} \times \overrightarrow{B}\right)_\theta = \mu_e H_0^2\left(\frac{\sin^2\phi}{r}\frac{\partial^2 v_2}{\partial z^2} + \cos^2\phi \frac{\partial^2 v_2}{\partial r^2} + \sin\phi\cos\phi \frac{\partial^2 v_2}{\partial r\partial z} + \frac{\sin\phi\cos\phi}{r}\frac{\partial^2 v_2}{\partial r\partial z}\right). \tag{14}$$

For shear harmonic waves, the displacement component $v_2(z)$ can be assumed as:

$$v_2(z) = f_2(z)J_1(kr)e^{i\omega t}. \tag{15}$$

Equations (8), (14) and (15) give:

$$\frac{d^2 f_2}{dz^2} + D_1\frac{dG_2}{dz}\frac{df_2}{dz} + D_2 f_2 = 0. \tag{16}$$

In equation (16),

$$D_1 = \frac{1}{DG_2}\left(\frac{dG_2}{dz} + \left(1 + \frac{1}{r}\right)\sin\phi\cos\phi J_1'(kr)\right), D = \left(1 + \frac{\mu_2\mu_e}{G_2}H_0^2\frac{\sin^2\phi}{r}\right),$$

$$D_2 = \frac{1}{DG_2}\left(k^2\left(\frac{\mu_2}{G_2} + \mu_2\mu_e H_0^2\cos^2\phi\right)J_1''(kr) + \omega^2\left(\frac{\rho_{1100}\rho_{2200} - \rho_{1200}^2}{\rho_{2200}}\right)\right), G_2 = \mu_2 - \frac{P_2}{2},$$

$$\mu_2 = \mu_{20}(1 + qz)^2, P_2 = P_{20}(1 + qz)^2, \rho_{1100} = \rho_{11001}(1 + qz)^2,$$

$$\rho_{1200} = \rho_{12001}(1 + qz)^2, \rho_{2200} = \rho_{22001}(1 + qz)^2, \xi_{20} = P_{20}/2\mu_{20}. \tag{17}$$

The solution of equations (16) and (15) give:
$$v_2(z) = \exp(-D_1 z/2)(C_3\cos(\chi_2 z) + C_4\sin(\chi_2 z))J_1(kr)e^{i\omega t},$$

Here, $\chi_2 = D_2^2, C_3, C_4$ are arbitrary constants. Substitution of above displacement component in equation (2) gives the following stress component in the medium under consideration:

$$
\begin{aligned}
(\sigma_{\theta z})_{M_2} = \, & C_3 \left(\left(\mu_{20}(1+qz)^2 \exp(-D_1 z/2) \right) \left(\cos(\chi_2 z)\chi_2 - (D_1/2)\sin(\chi_2 z)J_1(kr) \right) \right) \\
& + C_4 \left(\left(\mu_{20}(1+qz)^2 \exp(-D_1 z/2) \right) \left(\sin(\chi_2 z)\chi_2 - (D_1/2)\cos(\chi_2 z)J_1(kr) \right) \right)
\end{aligned}
$$

3.3 The lower poroelastic medium (z > 0)
In the lower heterogeneous poroelastic half space, let (u_3, v_3, w_3) and (U_3, V_3, W_3) be the solid and fluid displacement components, respectively. For shear wave in this space, the equation of motion (1) takes the following form

$$
\begin{aligned}
& \mu_3(z)\left(\frac{\partial^2 v_3}{\partial r^2} + \frac{1}{r}\frac{\partial v_3}{\partial r} - \frac{v_3}{r^2}\right) + \frac{\partial}{\partial z}\left(G_3 \frac{\partial v_3}{\partial z}\right) = \frac{\partial^2}{\partial t^2}(\rho_{11000} v_3 + \rho_{12000} V_3), \\
& 0 = \frac{\partial^2}{\partial t^2}(\rho_{12000} v_3 + \rho_{22000} V_3).
\end{aligned}
\tag{18}
$$

In equation (18), $\mu_3 = \mu_{30}(1 + l_1 z), G_3 = \mu_3(1 - P_3/2), P_3 = P_{30}(1 + l_2 z), \xi_{30} = P_{30}/2\mu_{30}. \rho_{11000} = \rho_{110001}(1 + l_2 z), \rho_{12000} = \rho_{120001}(1 + l_2 z), \rho_{22000} = \rho_{220001}(1 + l_2 z). \mu_{30}, P_{30}, \rho_{110001}, \rho_{120001}, \rho_{220001}$, are the initial values of the respective parameters. The non-zero stress components are $\sigma_{r\theta} = \mu_3(z)\left(\frac{\partial v_3}{\partial r} - \frac{v_3}{r}\right), \sigma_{z\theta} = \mu_3(z)\left(\frac{\partial v_3}{\partial z}\right)$. As in earlier cases, the solution of equation (18) $v_3(z)$ assumed as:

$$
v_3(z) = f_3(z)J_1(kr)e^{i\omega t}.
\tag{19}
$$

Equations (18) and (19) give:

$$
\frac{d^2 f_3}{dz^2} + \frac{1}{G_3}\frac{dG_3}{dz}\frac{df_3}{dz} - \frac{k^2 \mu_3}{G_3}\left(1 - \frac{c^2}{\beta_3^2}\right)f_3 = 0,
\tag{20}
$$

In equation (20), $\beta_3 = \left(\frac{\mu_{30}\rho_{2200}}{\rho_{11000}\rho_{22000} - \rho_{12000}^2}\right)^{1/2}$,

To make nonlinear equation (20) linear, assume that

$$
f_3(z) = \frac{f_{30}(z)}{\sqrt{G_{30}(1 + l_4 z)}},
\tag{21}
$$

here $f_{30}(z)$ is an arbitrary function, $G_{30} = \mu_{30}(1 - \xi_{30}), l_4 = l_1 - \xi_{30} l_3/1 - \xi_{30}.$ Then equation (20) reduces to:

$$
\frac{d^2 f_{30}}{dz^2} + \left[\frac{l_4^2}{4(1 + l_4 z)^2} - k_3^2\left(\frac{(1 + l_1 z)}{(1 + l_4 z)} - \frac{c^2(1 + l_2 z)}{\beta_3^2(1 + l_4 z)}\right)\right] = 0.
\tag{22}
$$

Introducing $\eta = \frac{2\chi_3 k_3}{l_4}(1 + l_4 z)$, here $\chi_3 = \left[\frac{1}{l_4}\left(l_1 - \frac{c^2}{\beta_3^2} l_2\right)\right]^{\frac{1}{2}}$, $k_3^2 = k^2 \mu_{30}/G_{30}$, and substituting $f_{30}(z) = \phi(\eta)$, equation (22) becomes:

$$\phi''(\eta) + \left(\frac{1}{4\eta^2} + \frac{R}{\eta} - \frac{1}{4}\right)\phi(\eta) = 0. \tag{23}$$

In equation (23), $R = \frac{k_3}{2\chi_3 l_4}\left[\left(\frac{c^2}{\beta_3^2} - 1\right) + \chi_3^2\right]$, $\chi_3 = \left[\frac{1}{l_4}\left(l_1 - \frac{l_2 c^2}{\beta_3^2}\right)\right]^{\frac{1}{2}}$, equation (23) is the well-known form of Whittaker equation (34). The solution of equation (23) $\phi(\eta) = C_5 W_{R,0}(\eta) + C_6 W_{-R,0}(-\eta)$. $W_{R,0}$, $W_{-R,0}$ are Whittaker functions, C_5, C_6 are arbitrary constants. For bounded solution, C_6 must be zero. In this case, the solution of equation (20) is:

$$f_3(z) = C_5\left[W_{R,0}\left(\frac{2\chi_3 k_3}{l_4}(1 + l_4 z)\right)\right]\frac{1}{\sqrt{G_{30}(1 + l_4 z)}}J_1(kr)e^{i\omega t}. \tag{24}$$

Using equations (24) and (19) in equation (2), the pertinent stress is given by:

$$(\sigma_{\theta z})_{M_3} = C_5\left[\left(\mu_3 l_4/G_{30}(1 + l_4 z)^{1/2}\right)\left[\left(l_4(R - z/2)/G_{30}(1 + l_4 z)^{1/2}\right)\right.\right.$$

$$W_R, 0\left(2\chi_3 k_3(1 + l_4 z)/l_4\right) + (R - 1/2)^2 W_{R-1,0}\left(2\chi_3 k_3(1 + l_4 z)/l_4\right)]$$

$$-\left(l_4/2G_{30}(1 + l_4 z)^{3/2}\right)W_{R,0}\left(2\chi_3 k_3(1 + l_4 z)/l_4\right)].$$

4. Boundary conditions and frequency equation
The boundary conditions at free boundary and two interfaces are prescribed as follows:

Assume that $z = -h_2$ is stress free if $(\sigma_{\theta z})_{M1} = 0$, at $z = -h_1$, the displacement and stress must be continuous, i.e. $|(\sigma_{\theta z})_{M_1} = (\sigma_{\theta z})_{M_2}, v_1 = v_2$, and at the plane of contact of the magneto poroelastic medium and poroelastic half space, i.e. at $z = 0$ the displacement and stress must be continuous, i.e. $(\sigma_{\theta z})_{M_2} = (\sigma_{\theta z})_{M_3}, v_2 = v_3$. These boundary conditions lead to the following system of homogeneous equations:

$$[A_{ij}][C_i] = [0]_{2,2} i = 1, 2, 3, 4, 5. \tag{25}$$

Equation (25) results in a system of five homogeneous equations in five arbitrary constants C_1, C_2, C_3, C_4, C_5. To obtain a non-trivial solution, the determinant of coefficients must be zero. Accordingly, the following frequency equation is obtained.

$$|A_{ij}| = 0, i, j = 1, 2, 3, 4, 5. \tag{26}$$

5. Particular cases
Case (i): In the sandwiched medium if the magnetic and porous properties are absent, then the problem is reduced to that of classical elasticity (Paswan *et al.*, 2017), and the frequency equation (26) is:

$$\begin{vmatrix} 0 & A_{10} & B_{10} \\ A_{20} & B_{20} & 0 \\ A_{30} & B_{30} & C_{30} \end{vmatrix} = 0. \tag{27}$$

In equation (27),

$$A_{10} = \chi_{10}J_1(kr), B_{10} = \frac{l_4(R)}{G_{30}}W_{R,0}\left(\frac{2k_3}{l_4}\right), A_{20} = S_3\chi_{10}J_1(kr), B_{20} = \mu_T S_2 kJ_1'(kr) - S_3\frac{E}{2}J_1(kr),$$

$$A_{30} = e^{\frac{-Eh}{2}}\sin(\chi_{10}h_1)J_1(kr), B_{30} = e^{\frac{-Eh_1}{2}}\cos(\chi_{10}h_1)J_1(kr), C_{30} = \left[W_{R,0}\left(\frac{2\chi_{30}k_3}{l_4}(1+l_4h_1)\right)\right]\frac{1}{\sqrt{G_{30}(1+l_4h_1)}}J_1(kr),$$

$$\chi_{10} = -\left(\frac{J_1''(kr)k^2}{\xi_{10}J_1(kr)} + \frac{J_1'(kr)k}{\xi_{10}rJ_1(kr)} + \omega^2\left(\frac{\rho_{110}}{\mu J_1(kr)}\right)\right).$$

$$\chi_{30} = \left[\frac{1}{l_4}\left(l_1 - \frac{c^2}{\beta_3^2}l_2\right)\right]^{\frac{1}{2}},$$

$$\beta_3 = \left(\frac{\mu_{30}}{\rho_{11000}}\right)^{1/2},$$

Case (ii): When the inhomogeneity parameter is absent, i.e. for upper layer $a \to 0, \mu_L = \mu_T = \mu$, for intermediate layer $q \to 0$, and for lower poroelastic half space $l_1 \to 0, l_2 \to 0, l_3 \to 0$, then the frequency equation (26) reduces to:

$$\begin{vmatrix} 0 & A_{11}\sin(\chi_{11}h_2) & 0 \\ 0 & A_{11}\sin(\chi_{11}h_1) & A_{21} \\ A_{31}\cos(\chi_2h_1) & A_{31}\sin(\chi_2h_1) & A_{41} \end{vmatrix}. \tag{28}$$

In equation (28),
$\quad A_{11} = \chi_{11}J_1(kr)$,

$$\chi_{11} = -\left(\frac{J_1''(kr)k^2}{\xi_{10}J_1(kr)} + \frac{J_1'(kr)k}{\xi_{10}rJ_1(kr)} - \frac{1}{\xi_{10}r^2} + \omega^2\left(\frac{\rho_{1100}\rho_{2200} - \rho_{1200}^2}{\xi_{10}\rho_{2200}\mu J_1(kr)}\right)\right),$$

$A_{21} = \mu_{20}e^{-D1h1/2}(\chi_2 - D_1h_1/2)J_1(kr), A_{31} = J_1(kr), A_{41} = e^{-D1h1/2}J_1(kr)$.
μ_{20}, D_1, are similar expressions as equation (17) with $q \to 0$.
Case (iii): Apart from the conditions of case (ii), if the initial stress is absent, i.e. for upper layer $P_1 \to 0, \xi_{10} \to 0$, for intermediate layer $P_2 \to 0, \xi_{20} \to 0, G_{20} \to \mu_{20}$, and for lower poroelastic half space $P_3 \to 0, \xi_{30} \to 0, G_{30} \to \mu_{30}, k_3 \to k$.
$\quad$Then the frequency equation (26) reduces to:

$$\begin{vmatrix} 0 & B_{12}\sin(\chi_{12}h_2) & 0 \\ 0 & B_{12}\sin(\chi_{12}h_1) & A_{22} \\ A_{32}\cos(\chi_2h_1) & A_{32}\sin(\chi_2h_1) & A_{42} \end{vmatrix} = 0. \tag{29}$$

In equation (29),
$\quad B_{12} = \chi_{12}J_1(kr)$,

$$\chi_{12} = \frac{J_1''(kr)k^2}{J_1(kr)} + \frac{J_1'(kr)k}{rJ_1(kr)} - \frac{1}{r^2} + \omega^2\left(\frac{\rho_{1100}\rho_{2200} - \rho_{1200}^2}{\rho_{2200}\mu J_1(kr)}\right),$$

The other expressions for equation (29) are similar to that of equation (28) with $P_2 \to 0$.

6. Numerical results

For numerical process, the following materials are used.

The material parameters for upper self-reinforced medium (Chattaraj and Samal, 2013) (Say Mat-I), are:

$$\mu^T = 2.46 \times 10^9 N/m^2, \; \mu_L = 5.66 \times 10^9 N/m^2, \; \rho_{110} = 1.92637 \times 10^3 kg/m^3$$

$$\rho_{120} = -0.002137 \times 10^3 kg/m^3, \; \rho_{220} = 0.215337 \times 10^3 kg/m^3$$

The intermediate magneto poroelastic medium is assumed to be sandstone saturated with kerosene (Yew and Jogi, 1976) (Say Mat-II). The values in this case are $\mu_{20} = 70199 \times 10^{10} N/m^2$, $\rho_{1100} = 24152 \times 10^3 kg/m^3$, $\rho_{1200} = -300 kg/m^3$, $\rho_{2200} = 500 \times 103 kg/m^3$.

The lower half-space is assumed to be sandstone saturated with water (Fatt, 1959) (Say Mat-III). The values in this case are $\mu_{30} = 0.92 \times 10^{10} N/m^2$, $\rho_{11000} = 1.9002 \times 10^3 kg/m^3$, $\rho_{12000} = 0$, $\rho_{22000} = 0.2268 \times 10^3 kg/m^3$.

Using these values in the frequency equation, the implicit relation between the phase velocity and wavenumber is obtained. Phase velocity is computed against wavenumber. The magneto-poroelastic coupling factor is taken to be $\mu_e H_0^2/N = 0.01$ (Balu *et al.*, 2017). Moreover, the values for ϕ, kr are taken to be $\pi/6, 5$ (Chattopadhyay *et al.*, 2012) and the other parameter values are in various cases specified in Table 1. The values are computed using the bisection method implemented in MATLAB, and the results are depicted in Figures 2-6.

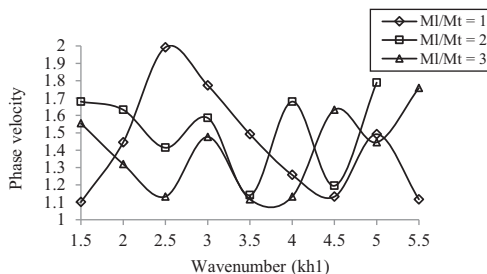|	ah_1	l_1h_1	l_2h_1	l_3h_1	qh_1	ξ_{10}	ξ_{20}	ξ_{30}	h_2/h_1	μ_l/μ_t
Figure 2	0.4	0.5	0.4	0.3	0.4	0.4	0.3	0.2	–	1.2
Figure 3	–	0.5	0.4	0.3	0.4	0.4	0.3	0.2	1.6	1.2
Figure 4	–	0.5	0.4	0.3	0.4	0.4	0.3	0.2	1.6	–
Figure 5	0.4	0.5	0.4	0.3	–	–	0.3	0.2	1.6	1.2
Figure 6	0.4	0.5	0.4	0.3	0.4	–	0.3	0.2	1.6	1.2

Table 1.
Parameter values

Figure 2.
Variation of phase velocity with wavenumber for various values (μ_L/μ_T) in upper reinforcement medium

Figure 3.
Variation of phase
velocity with
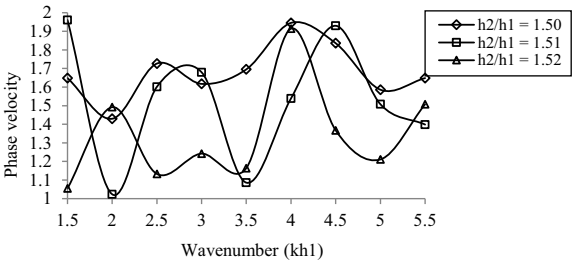wavenumber for
various ratios (h_2/h_1)

Figure 4.
Variation of phase
velocity with
wavenumber for
various values of
inhomogeneity
parameter (ah_1) in
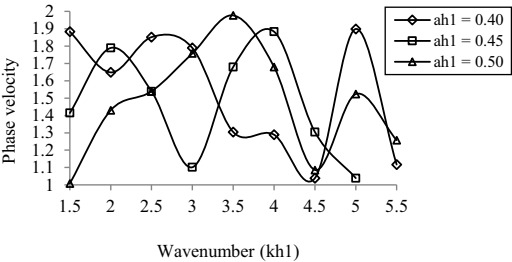upper reinforcement
medium

Figure 5.
Variation of phase
velocity with
wavenumber for
various values of
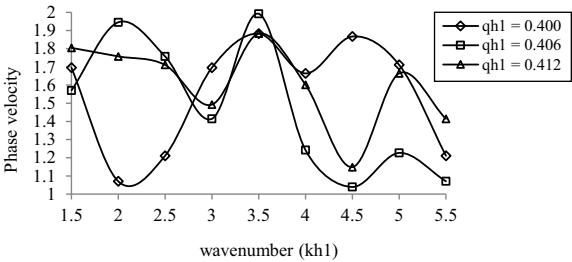inhomogeneity
parameter (qh_1) in
magneto poroelastic
medium

Figure 6.
Variation of phase
velocity with
wavenumber for
various values of
heterogeneity
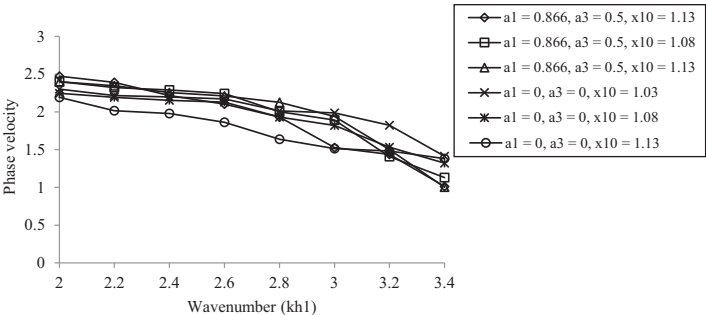parameter (χ_{10}) in
elastic medium

Figure 2 depicts the variation of phase velocity with wavenumber for different values of
(μ_L/μ_T) in upper reinforcement medium. From this figure, it is clear that, as wavenumber
increases, phase velocity, in general, decreases. The notations M1/Mt = 1, M1/Mt = 2, M1/Mt
= 3 used in the Figure 2 represent the different values of (μ^L, μ^T). Figure 3 depicts the
variation of phase velocity with wavenumber for different values of (h_2/h_1). From Figure 3, it
is seen that as wavenumber increases, phase velocity, in general, decreases. Figure 4 depicts
the variation of phase velocity with wavenumber for various inhomogeneity parameter
values in upper reinforcement medium. From this figure, it is observed that, as wavenumber
increases, phase velocity, in general, decreases for all values of inhomogeneity parameter
(ah_1). Figure 5 depicts the variation of phase velocity with wavenumber for various
inhomogeneity parameter values in magneto poroelastic medium. From this figure, it is clear
that, as wavenumber increases, phase velocity, in general, increases when the
inhomogeneity parameter $qh_1 = 0.400$, whereas when $qh_1 = 0.406, 0.412$, phase velocity, in
general, decreases. For the validation, the results are computed for particular case (i). Phase
velocity is computed against the wavenumber for various heterogeneity parameters in
presence of reinforcement and free from reinforcement. The results are presented in
Figure 6. In Figure 6, Curves 1, 2 and 3 pertaining to presence of reinforcement, whereas the
Curves 4, 5 and 6 pertaining to free from reinforcement for the steel (Paswan *et al.*, 2017).
From this figure, it is clear that as wavenumber increases phase velocity decreases as the
values of heterogeneity parameter (χ_{10}) increases in both reinforced and reinforced free
cases. These results are in agreement with that of the paper (Paswan *et al.*, 2017).

7. Conclusion

Shear wave propagation in magneto poroelastic medium embedded between a self-
reinforced medium and poroelastic half space are investigated in presence of initial stress,
and inhomogeneity parameter. For heterogeneous poroelastic half space, the Whittaker's
solution is obtained. From the numerical results, it is observed that heterogeneity parameter,
inhomogeneity parameter and reinforcement parameter have significant influences on the
wave characteristics. In addition, frequency equation is discussed in absence of
inhomogeneity and initial stress. For the validation purpose, numerical results are also
computed for a particular case.

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